Harmonic structures: analysis and application of two layers of three-part chord structures in perfect fifths

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Abstract

This document describes the analysis and application of harmonic structures, based on two layers of three-part chords in perfect fifths. The resulting chord structures are classified along a consonance-dissonance scale and two score examples are discussed. Document history: update April 2010.

1 Introduction

The technique, described in this document, deals with the combination of two layers of harmonic structures in perfect 5ths. These combined structures were used by Béla Bartók in the second movement of his Piano Concerto No. 2 (here discussed as an application example). This technique can be used to create a specific mood (mystical, tranquil) with controllable degree of dissonance. Also, a simple counterpoint is created by using independent root movement in the two layers. Here, we will discuss the classification of the set of possible chord structures and their application in musical composition or arrangements.

2 Building the chord structures

The chord structures are created by combining two harmonic layers, where each layer contains 3-part chords in open voicing perfect 5ths. This combination yields twelve possible chord structures, shown in Fig. 1. The chord root in the lower layer R_1 is always C and the root in the upper layer structure R_2 passes through all the steps of the 12-pitch chromatic scale.

The representation of the combined chord structures as a tone disc is shown in Fig. 2. The tone disc retains the cyclic character of the 12-pitch chromatic scale: the pitches are numbered clockwise from the top: $C = 0, C\sharp = 1, \ldots, B = 11$. Octave transposition is mathematically equivalent to the modulo 12 operation (a full rotation in the tone disc). The tone disc representation helps in graphically identifying symmetry and dissonance in atonal chord structures.

3 Classification of the chord structures

The twelve possible combined chord structures are shown in Table 1, and labeled S_0 to S_{11} (first column). Three analysis tools will be used for classification of these intervallic structures.

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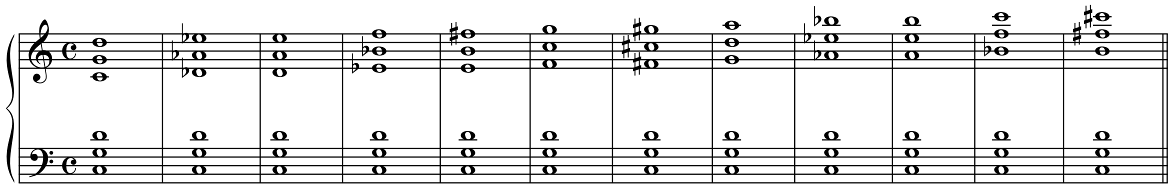
Root: $R_2 = C$, $D\flat$, ... B



Root: $R_1 = C$

Structure: S_0 S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11}

Figure 1: The twelve 6-part chord structures $S_0, S_1, \ldots, S_{11}$, resulting from the combination of two harmony layers with 3-part chords in open voicing perfect fifths. Constant lower layer root $R_1 = C$ and variable upper layer root $R_2 = C, D\flat, \ldots, B$.

Table 1: The twelve 6-part chord structures, resulting from the combination of two layers with 3-part chords in perfect fifths. The second and third column show the classification according to Pitch-Class set theory by Forte [1], the fifth column shows the tension level classification according to Ulehla [3] and the last column shows the tonal equivalent (when it exists).

Chord label	**Pitch-Class set**	**Root**	**Interval vector**	**Tension level**	**Tonal equivalent**
S_0	3-9	O_0	[010020]	2	$S_{\text{sus4}}, S^{\text{no3}}_{\text{add9}}$
S_1	6-Z38	O_0	[421242]	9	
S_2	5-35	O_0	[032140]	1	S^{11}_{m7}
S_3	6-32	O_{10}	[143250]	8	$S^{9/11}_{m7}$
S_4	6-Z26	O_{11}	[232341]	9	$S^{9/\sharp 11}_{\Delta 7}$
S_5	4-23	O_0	[021030]	1	S^{11}_{m7}
S_6	6-7	O_0	[420243]	9	
S_7	4-23	O_7	[021030]	1	S^{add11}_{m}
S_8	6-Z26	O_7	[232341]	9	$S^{9/\sharp 5}_{\Delta 7}$
S_9	6-32	O_7	[143250]	8	$S^{6}_{\Delta 7}$ major hexachord
S_{10}	5-35	O_{10}	[032140]	1	S^{11}_{m7}
S_{11}	6-Z38	O_{11}	[421242]	9	

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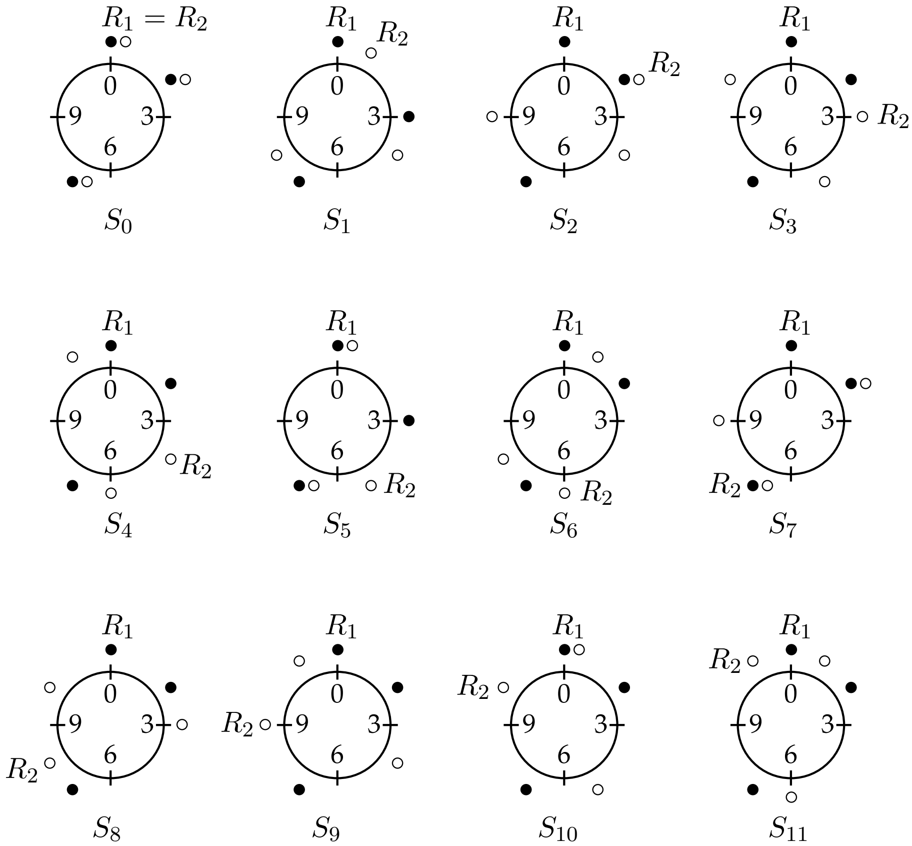

Figure 2: Tone disc representation of the twelve 6-part chords in perfect fifths, labeled S_0 to S_{11}. Symbols: •: pitches in Layer 1 (lower layer), ◦: pitches in Layer 2 (upper layer), $R_{1,2}$: layer roots.

3.1 Pitch-Class sets Prime Forms and Interval Vectors (A. Forte)

The second column in Table 1 shows the Pitch-Class set (PC set) *Prime Form* number, using the numbering system by Forte [1].[1] Note the symmetry in the Pitch-Class sets with respect to S_0, i.e., there are only six possibly solutions (one 3-, 4- and 5-pitch set each, and three 6-pitch sets) with equal sets for S_1 and S_{11}, S_2 and S_{10}, etc. Column 3 shows the root of the PC class; O_{10} means, that the PC class is in the original form (non-inverted, clockwise orientation in the tone disc diagrams) with $B\flat$ (pitch number 10) as the root. There are no inverted PC sets in the table.

The interval content is shown in the fourth column of Table 1 as an *Interval Vector*: this is a row vector with six elements $[n_1 n_2 n_3 n_4 n_5 n_6]$ (no spaces between elements), with n_i indicating the total number of interval classes with i (or $12 - i$, the octave inversion) semitones; $i = 1$ is the semitone, $i = 2$ the major second or whole tone, and so forth until $i = 6$ the augmented 4th or diminished 5th, also called the tritone. So, for example, we find for PC set 4-14 the interval vector [111120] (i.e., no tritone, two perfect fourths or fifths and one occurrence of the other interval categories).

[1] The Website www.fransabsil.nl has an interactive graphical toolkit for inspecting and analysing pitch-class sets and interval vectors, using the Forte numbering system. See under the menu label Archive.

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3.2 Degree of consonance-dissonance according to Paul Hindemith

Note that in the atonal PC set system the intervals and their octave inversions are considered equivalent. However, Hindemith [2] distinguishes a dissonance ranking where the octave inversion is the more dissonant interval. This leads to the following consonant-to-dissonant ranking:

$$\text{Hindemith consonant to dissonant ranking} : 0i, 12i, 7i, 5i, 4i, 8i, 3i, 9i, 2i, 10i, i, 11i, 6i, \quad (1)$$

where i is the semitone interval, and all other intervals are expressed as integer multiples of the semitone. So, we see first the perfect consonants (unison, octave, perfect 5th and 4th), then the imperfect consonances (major and minor 3rds and 6ths), and finally the dissonants (major and minor 2nds and 7ths, and the augmented 4th or diminished 5th). In Hindemith's ranking the major 3rd ($4i$) is considered more consonant than the minor 6th ($8i$), the major 2nd ($2i$) more consonant than the minor 7th ($10i$), etc.

This dissonance ranking will affect the tension level of the chord structures in Table 1, in the sense that S_1 has a higher tension level than S_{11}. The same holds for S_3 compared to S_9 and S_4 compared to S_8. In general this will alter the symmetry property in the sense that the tension level is somewhat lower if the root of the upper layer structure lies below the root of the lower layer (see Fig. 1, where structures more to the right are more consonant than their equivalent on the left). This ranking effect is indicated in the PC set analysis examples below as the Hindemith correction.

3.3 Degree of vertical tension according to Ludmilla Ulehla

In Ulehla's book, Chapter 19, *The Control of Dissonance*, [3] there is a dissonance ranking system for chord structures in modern music. This ranking may be used to analyse or create tension curves in musical phrases. The ranking system originally uses a graphical representation on a musical staff; the tension level line may have 10 positions (5 on the staff lines, 4 between lines and the highest tension level above the staff).

There is a useful correspondence between the tension levels, as defined by Ulehla and the Interval Vectors, as introduced by Forte. An overview of this correspondence is shown in Table 2. Keep in mind that n_1, n_2 and n_6 represent the dissonant intervals, n_3 and n_4 the imperfect consonances, and n_5 the perfect consonances.

The tension level according to this classification is indicated in the fifth column of Table 1. Note that the combined perfect fifth chord structures are of either low (1 and 2) or very high (8 and 9) tension level.

4 Two examples

Below, two examples will be discussed: one from the the 20th century classical musical repertoire, and one arrangement of a jazz standard. We will label the chord structures and analyse the dissonance and tension curves of these fragments.

4.1 Béla Bartók: Piano Concerto no. 2, Mvt. 2

Fig. 3 shows the introduction to the second, slow movement (Adagio, 66-69 BPM) played by non-vibrato, muted strings at pianissimo (*pp*) dynamic level.

Table 2: The correspondence between the tension levels as defined by Ulehla [3] and the interval vector, introduced by Forte [1].

Tension level	**Description of chord structure**	**Interval vector and conditions**
1	triads and inversions	[001110]
2	minor 7th chords	[012120],
	with 9th and/or11th	$[n_1 n_2 n_3 n_4 n_5 0]$ with
	extensions	$n_1 = \{0, 1\}$ and n_2, n_3, n_5 high
3	major 7th chords	[101220]
4	concordant (non-triadic) intervallic structures	$[0 n_2 n_3 n_4 n_5 0]$
5	diminished triad, half-diminished 7th chord	[002001], [012111]
	whole-tone combinations	$[0 n_2 0 n_4 0 n_6]$
6	dominant 7th chord,	[012111],
	including major 9th extension	[032221]
7	dominant 7th chord	[114112],
	with minor 9th and other extensions	$[n_1 n_2 n_3 n_4 n_5 n_6]$ with $n_6 \geq 2$
8	discordant intervallic structures	$[n_1 n_2 n_3 n_4 n_5 0]$,
	without a tritone	with $n_1 > 1$
9	discordant intervallic structures including	$[n_1 n_2 n_3 n_4 n_5 n_6]$
	(one or more) tritone(s) and any	with $n_6 \geq 1$
	combination of minor 2nds or major 7ths	$n_1 \geq 1$
10	chords of seven or more tones with	$[n_1 n_2 n_3 n_4 n_5 n_6]$ with $\sum_i n_i \geq 21$
	maximum discordant level	and n_1, n_2 and n_6 high

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Figure 3: Example 1: Béla Bartók, *Piano Concerto No. 2*, Mvt. 2 *Adagio*, 66-69 BPM, mm. 1–24 (Philharmonia Partituren, No. 306, Universal Edition, Wien–London). Pitch-Class set labels are shown above the staff. The tension level contour line is shown below the staff (reduced vertical scale). The thin line represents the Hindemith dissonance ranking correction.

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The figure demonstrates a number of characteristics of this opening fragment: motion in both harmonic layers mostly is stepwise, and in contrary motion. Next, within a bar there is frequent use of the symmetry property of the chord structures, with a tendency to move from the higher tension level to the lower level harmonic structure (see the Hindemith correction lines in the graph). For example, m. 3 has the sequence $S_9 - S_3$ for the PC set 6-32, but is an exception in the sense that the tension level increases between these two chords. Another example is m. 8 where we see the $S_{10} - S_2$ sequence of set 5-35. The tension line is shown below the staff at reduced vertical scale; note that the opening measure has high tension level (8) and the closing measures have low tension level (1). In total there are two occurrences of level 9 (m. 8, beat 1, and m. 11, 3rd beat). Neither the pair $S_1 - S_{11}$ (PC set 6Z-38) nor S_6 (PC set 6-7) have been used. Note how the tension level rises for suspended (m. 4, 9, 16 and 19) chords; a careful handling of tension increase, followed by a decrease.

The same technique returns three times in this movement: see m. 54–61 of the opening Adagio section, and m. 1–9 (where the strings are playing tremolo) and m. 32–37 of the closing Adagio. The second and third return are moving from tension level 1 to 9 (towards increasing dissonance), whereas the last fragment moves from tension level 9 to 1 (i.e., tension release). There is a single occurrence of the highly dissonant PC set 6-Z38 in m. 7 of the closing Adagio.

4.2 I'll Remember April (G. de Paul): arrangement for studio orchestra

The score example in Fig. 4 is the slow introduction (60 BPM) to my arrangement of this jazz standard for a studio orchestra workshop. It is played by tremolo strings at mezzo piano (*mp*) dynamic level, with timpani and celesta fills on the fermata chords in m. 4 and 8 (not shown here). The first violin lead carries the melody of the song.

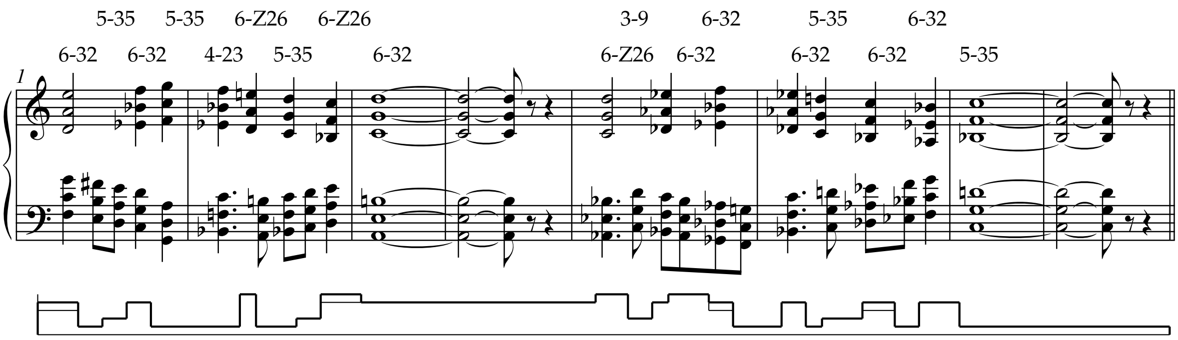

Figure 4: Example 2: Gene de Paul, *I'll Remember April*, Arrangement for orchestra, Introduction, 60 BPM, mm. 1–8 (NOS Music Library, Hilversum). Pitch-class set labels are indicated above the staff (odd beats in lower line, even beats in upper line) The tension level contour line is shown below the staff (reduced vertical scale). The thin line represents the Hindemith dissonance ranking correction.

This introduction uses the same PC sets as the Bartók example, and applies the same overall tension level tendency, i.e., starting at high level, and ending at low tension level (towards a tension release). There is also the contrary, stepwise motion in the two layers. The symmetry property was used occasionally, with a preference for the lower tension level chord (i.e., more frequent use of S_9 than S_3 in the case of PC set 6-32).

5 Conclusion

This harmonic technique is particularly useful:

- in slow tempos (the examples are Adagio's with 60-70 BPM).
- for introductions, codas and transitions. Or, as a harmonic background for a free improvisation.
- when maintaining an open voicing in each layer. The mood of this setting changes significantly when using closed voincing in either layer.
- when applying 2-part counterpoint to the roots of the layers. Most of the counterpoint will be in contrary, stepwise motion.
- for a medium to large string section (both examples are written for divisi strings), playing softly, muted or in tremolo style.
- for low woodwinds (e.g., bassoons and bass clarinet in the lower layer, clarinets, alto flute or flute in the upper layer). Some brass sounds, such as bucket mute trombones, may also be used in the lower layer.
- for synthesizer pads (not necessarily string sounds).

Applying this technique, a specific mood can be created with a controllable degree of dissonance and tension. Find more examples of this harmonic technique in the classical music repertoire or try an example yourself.

References

[1] Allen Forte. *The Structure of Atonal Music*. Yale University Press, New Haven and London, 1973. ISBN 0-300-02120-8. ix + 224 pp.

[2] Paul Hindemith. *Unterweisung im Tonsatz: theoretischer Teil*, volume I, ED3600. Edition Schott, Mainz, 1940. ISBN 3-7957-1690-4. 260 pp. (in German).

[3] Ludmilla Ulehla. *Contemporary Harmony; Romanticism through the Twelve-Tone Row*. Number Order # 11400. Advance Music, USA, 1994. x + 534 pp.

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